Introduction to Systems and Mathematics*

The Model introduced in *Introduction to Systems* (*IS*) (see http://www.marionbrady.com/IntroductiontoSystems.asp) allows learners to apply its insights and organizing scheme across the whole realm of knowledge. Many of its investigations require the use of mathematics in gathering, quantifying and analyzing information. Refer to *IS* for explanations of terms such as "Target Area."

IS isn't, of course, a substitute for classes that focus entirely on math skills, but it can provide opportunities for practical application of those skills, with beneficial effects on learner attitude toward mathematics. An example of how this works out in practice is at http://www.marionbrady.com/IntroSystems/DrWilliamWebb-Testimonial.pdf. Additional investigations given in this paper are centered more directly on mathematics, to show the range of possibilities. Activities of this sort, with learners out of their seats directly confronting the real world, are far more effective than any conventional learning activity.

As with other *IS* investigations, the teacher or mentor must be careful not to "short circuit" the learning process. Struggling with problem solutions takes time, but when the aim is intellectual growth, failing and starting over is time well spent. Opinions of authority figures should be minimal—offered only if essential to learning.

We suggest learners work in teams where possible. (Note that additional math resources focusing on reality are at http://www.realworldmath.org).

Here's a comment Edric Cane, author of perceptive math education book *Teaching to Intuition* (2013, Dog Ear Publishing) recently made to Marion Brady in an email:

"Just measuring is a rather restrictive view of what the tool of math can contribute to one's examination of a school. Ratios, for instance, are still central to mathematics but immediately look beyond mere numbers and offer the opportunity to analyze reality in terms of other perspectives: the ratio of teachers to students, of adults to teachers, of pages in a math book to school days and math periods in a year, of classroom area to open space area, such numbers can be made to speak and give meaning."

Dr. Cane and others have also suggested exercises in **estimating** (distances, quantities, ratios, etc.) to develop useful math skills that aren't a usual part of mathematics education. For example, the initial "RHRN" investigation—preparing a diagram of the target area—could first be done as a sketch, filled in with estimates of distances and spaces. The actual measurements that follow will help test and refine learner's estimating abilities.

Comments for teachers or mentors are marked "**TN.**"

Investigation: Measuring distances

A traditional way of determining distance is "pacing it off." If you know the length covered by your average step ("pace"), then walking the distance, counting your steps and multiplying that number by your pace length will give you an approximation of the distance.

1. ***Calculate the length of your pace, by pacing off a measured distance.***
2. ***Choose an unknown distance to measure, such as the length of a long corridor or the distance between two outdoor objects such as fences.***
3. ***Record your estimate of the distance before pacing it off.***
4. ***Pace off the unknown length, and calculate the distance. If you're working in a group, obtain and record results for each person in the group. If you're working alone, pace off the distance three or more times, and record the results.***
5. ***Compare your results (both preliminary estimates and results of pacing) with measurements made with a long tape measure. How accurate were the estimates? How accurate is pacing? What are typical amounts of error?***
6. ***If possible, average pacing results from several people. Is the average more accurate than that from only one person? If more than one group is doing pacing measurements, compare group averages.***
7. ***Are the estimated and paced results "biased," consistently too short, too long, or are they about right (i.e. "unbiased")?***
8. ***Discuss: What are possible causes of estimation error? Measurement error?***

TN: This is a practical introduction to the principles associated with measurements of any kind, and dovetails well with the "Target Area" investigations in *IS*. You will require a 50- or 100-foot (or comparable metric) tape measure, to determine both the preliminary known distance (e.g., length of the room) used to determine pace length, and the "unknown" distance used for the main investigation. Depending on the situation, you might choose to do the taped measurements yourself, prior to learner involvement, rather than having learners do it.

Some of the concepts and ideas to be developed:

- ***Every* measurement is an approximation;** errors vary with method of measurement and are affected by human factors such as how carefully measurements are made. Measurement accuracy will be affected by mistakes as well as random errors and errors of technique. For example, if the distance being measured by tape is longer than the tape, and the measuring path zigzags a bit, the measured value will be longer than the true distance.
- **Precision vs. accuracy:** If the results of pacing of a large group of learners are averaged, a result can be expressed very *precisely* (e.g. 136 feet 4-3/8 inches), but this precision can be misleading; the answer is unlikely to be very *accurate*.
- **Statistical variation:** If multiple results of pacing are plotted, the distribution of estimates will cluster around an average value (which, of course, may not be very accurate). An intuitive understanding of this distribution by learners could include calculation of "average error" and noting "outliers." Possible reasons for outliers include a mistake in calculation, or a learner's actual steps across the unknown

distance being significantly longer or shorter than the pace length determined earlier.

- **Error characteristics:** Biased data, accuracy of method used (e.g., the length of pace will vary from pace to pace, the original measurement of pace length may be inaccurate, mistakes in calculation, even manufacturing error in tape length or user's error marking off when multiple tape lengths are used).

Investigation: Practical math in horticulture

The recommended application rate for a standard brand of lawn fertilizer is 2.8 pounds for every 1000 square feet of lawn, applied every six weeks during the growing season. The fertilizer comes in 42-pound bags. How many bags will it take to fertilize the grass in your Target Area for one year? If each bag costs $28.98, what's the total fertilizer cost per year? Document your calculations.

Interview the person or persons who take care of your Target Area grass. Find out if they fertilize the grass, and if so, what kind of fertilizer they use, how much they use, how much it costs, and how often they fertilize. Compare the cost and application rate actually used with what you've calculated.

Grass grows successfully in many places without being fertilized. How could you determine if fertilizing is worth its cost on the Target Area grass?

TN: Of course, this activity cannot be done in some geographic locations, and seasonal variation may be a problem (e.g. ground covered by snow). This exercise must be done at some point after students have completed their survey of the Target Area described on pages 12-13 of *IS*. Additional measurements may be required to determine the area of the grass in the Target Area, of course. Incidentally, all the numbers in the first part of this investigation are actual information from a bag of Vigoro® fertilizer.

Investigation: Slope

"Slope," in mathematical terms, is the measurement of the rate at which an inclined line rises or falls. If a road sign says, at the top of a mountain, "Danger: 12%" grade ahead; truckers use low gear," this tells you that the roadway has a slope of twelve feet for every hundred feet of horizontal motion—a very steep roadway. (For roads, "grade" is another name for "slope." Standards for Interstate highways allow a maximum of 6% grade.)

Slope is sometimes given in percentage, as in this example, but usually is expressed as a number, such as "0.12" (12%). To find the slope, divide the "rise" or "fall" (vertical distance) by the "run," (horizontal distance). If the rise and run are equal, the slope is "1."

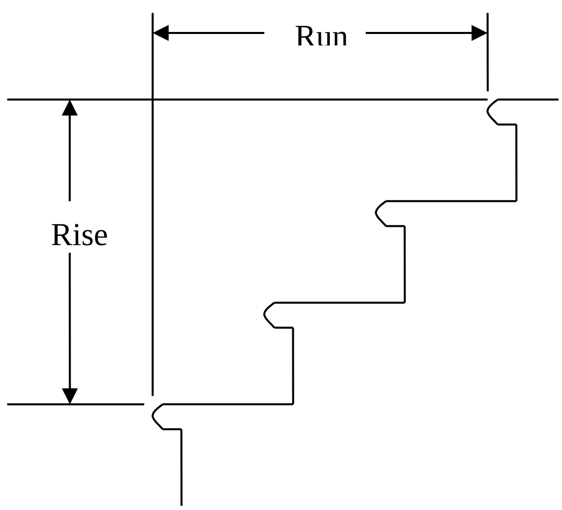
(1) ***Measure and calculate the slope of the nearest stairway. In your opinion, is the slope comfortable for most people climbing the stair, or is the slope too great or too small? (Record your measurements, calculations, results, and opinion.)***

(2) ***Find a nearby sloped roof, and figure out how to measure its slope without climbing onto the roof. Document your procedure.***

TN: The concept of slope is an important one in mathematics—one of the foundations of analytical geometry and calculus.

Finding the slope of a stairway is complicated by tread overhang and possible rounding of the leading edge of each tread. These must be taken into account in making measurements. (See figure below.) Measurements will be more accurate if they include several stairway steps instead of just one. A plumb bob and/or a carpenter's level, if available, will increase accuracy.

Worth discussing: If a photograph of a roof is used to calculate its slope, what are possible sources of error? (Main problems: The apparent roof slope will vary with camera position, and reference line for "horizontal" may be missing.) Here's a clue to one way to find a roof's slope: If the roof angle (α) from horizontal can be measured with reasonable accuracy, then *slope = tan α.*



Investigation: Indirect measurement

How tall is the tallest object (e.g. flagpole) in or near the Target Area? How high is the peak of the roof of the Target Area building? Decide on a procedure for determining the height of an object when it can't be measured directly, then make the measurements. In your journal, use words and diagrams to describe your procedure and results.

TN: Students should be left to struggle with this "Right Here, Right Now" challenge, and only helped with minimal suggestions, if they fail to come up with ideas. Measurements may be done several ways. One way involving simple trigonometry is to lay out a measured horizontal distance (d) from the base of the pole of the pole or object, and use some way of measuring (at this point) the upward angle (α) from horizontal to the top of the object. The height (H) can be calculated by multiplying the distance (d) by the tangent of the angle (tan α):

$$H = d * \tan \alpha$$

Sighting along pins stuck in a piece of corrugated cardboard is one possible way of measuring the angle, but accurate measurement will require some way of marking true horizontal on the cardboard when the sight alignment is made. Holding a short bubble level against the cardboard, centering the bubble and marking a line is one way of doing this. The angle between the line set by the pins and this horizontal line can then be measured with an ordinary protractor.

As part of an introduction to trigonometry in the eighth grade, the author was given this flag-pole problem, and built, with the help of his father, a simple instrument consisting of a protractor mounted on a board, with a clear plastic arm pivoting at the center point of the protractor. A pair of pointers made of small nails embedded in the clear plastic arm provided sights, and the arm was scribed so the sighting angle could be read off the protractor. A low-cost bubble level from the dime store was screwed to the board, set so when the bubble was centered, zero degrees on the protractor was horizontal.

Another, easier way that will work in some places is to measure (on a sunny day) the horizontal length of the pole’s shadow, and compare it with the horizontal shadow length of a vertical rod or stick of known length, with one end of the rod placed on level ground in sunlight. The triangles formed by the height, shadow length, and tip shadow path are “similar,” in the geometric sense, so the ratio of height to shadow length will be the same for both. Note that if the ground isn’t level for both the flagpole and the rod, measuring the shadow lengths along the ground may cause calculation errors. Another error might result from apparent motion of the sun unless the measurements of the two shadows are done within a few minutes of each other.

As a useful expansion of this investigation, students can determine the volume of the largest room (e.g. auditorium or gymnasium) in the Target Area building, and/or the volume of the entire building. These calculations, of course, require measurement of height. Knowing the volume of air heated or cooled is a significant Target Area element.

Howard Brady, September 2011 (Minor revision, September 2013, Added “Distance measurement,” June 2014)

Description of and links to course materials for *Introduction to Systems:*
http://www.marionbrady.com/IntroductiontoSystems.asp